Two-Dimensional and Axisymmetric Bifurcated Channel Flow

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Abstract

BODY-fitted coordinate techniques coupled with standard finite difference procedures are used to solve irregularly shaped bifurcated channel flow numerically. The approach is applicable to two-dimensional and axisymmetric laminar and turbulent flow. A two-equation eddy viscosity model is used to model the Reynolds stresses. Solutions were obtained for a range of Reynolds numbers of 1350 to 750,000. It was found that the first-order terms generated by the transformation process and those generated from the Reynolds stresses must be individually upwind differenced to obtain a converged solution. The metric coefficients generated by the transformation creates the need for large core storage or off-line storage with large input/output time.

Contents

Air moving through a turbofan engine passes through a channel located between the fan section and the high pressure section. Before entering the high pressure section, a splitter plate divides the flow into two streams (Fig. 1). It is important for engine performance that the fluid remain attached. Thus, it is important during the engine design stage to be able to determine the flow characteristics within the passageway for arbitrary geometries over a range of Reynolds numbers. This may be accomplished by constructing a computational model capable of handling irregularly shaped flow regions with laminar or turbulent flow.

The model developed here uses a vorticity-stream function formulation of the governing equations for two-dimensional and axisymmetrical flows with a two-equation eddy viscosity model for turbulence closure. A body-fitted coordinate system[1-3] is used to treat the arbitrary geometry.

Generation of the Coordinate System

The curvilinear coordinate system is generated as the solution of the following set of Poisson equations,

$$\frac{\partial^2 \xi}{\partial x^2} + \frac{\partial^2 \xi}{\partial y^2} + \frac{1}{y}\frac{\partial \xi}{\partial y} = P(\xi,\eta) \tag{1a}$$

$$\frac{\partial^2 \eta}{\partial x^2} + \frac{\partial^2 \eta}{\partial y^2} + \frac{1}{y}\frac{\partial \eta}{\partial y} = Q(\xi,\eta) \tag{1b}$$

where P and Q control the grid spacing in the physical plane. The two-dimensional form is recovered by deleting the first derivative terms. Solution of these equations is easier in the transformation plane, where the equations become

$$\alpha\frac{\partial^2 x}{\partial \xi^2} + 2\beta\frac{\partial^2 x}{\partial \xi \partial \eta} + \gamma\frac{\partial^2 x}{\partial \eta^2} = P\frac{\partial x}{\partial \xi} + Q\frac{\partial x}{\partial \eta} \tag{2a}$$

$$\alpha\frac{\partial^2 x}{\partial \xi^2} + 2\beta\frac{\partial^2 y}{\partial \xi \partial \eta} + \gamma\frac{\partial^2 y}{\partial \eta^2} = P\frac{\partial y}{\partial \xi} + Q\frac{\partial y}{\partial \eta} \tag{2b}$$

The differences between a two-dimensional and axisymmetric coordinate system are all incorporated in the coefficients α, β, γ, P, and Q (Ref. 3). Figure 2 shows the curvilinear coordinate system generated by solving Eqs. (2). Notice that coordinate lines are concentrated near the walls where velocity gradients are large. Coordinate line concentration is governed by the values of P and Q; P was set to zero and Q was determined from the limit of Eqs. (1) as x goes to zero.[4] In the transformed plane, this is a uniform rectangular region.

The flow is modeled using a vorticity-stream function formulation with the Kolmogorov-Saffman two-equation eddy viscosity model[5] to model the Reynolds stress terms. The governing equations are as follows (Z represents vorticity):

$$\frac{\partial Z}{\partial \tau} + U\left[\overset{(1)}{\xi_x\frac{\partial Z}{\partial \xi}} + \overset{(2)}{\eta_x\frac{\partial Z}{\partial \eta}}\right] + V\left[\overset{(3)}{\xi_y\frac{\partial Z}{\partial \xi}} + \overset{(4)}{\eta_y\frac{\partial Z}{\partial \eta}}\right]$$
$$+\frac{Z}{Y}\left[\frac{1}{YR_{\text{eff}}} - V\right] = \frac{1}{R_{\text{eff}}}\left[\alpha\frac{\partial^2 Z}{\partial \xi^2} + 2\beta\frac{\partial^2 Z}{\partial \xi\partial \eta} + \gamma\frac{\partial^2 Z}{\partial \eta^2}\right.$$
$$\left.+\overset{(5)}{P\frac{\partial Z}{\partial \xi}} + \overset{(6)}{Q\frac{\partial Z}{\partial \eta}}\right] + 4\left[\xi_x\frac{\partial}{\partial \xi}\left(\frac{1}{2R_T}\right) + \eta_x\frac{\partial}{\partial \eta}\left(\frac{1}{2R_T}\right)\right]$$
$$\times\left[\overset{(7)}{\xi_x\frac{\partial Z}{\partial \xi}} + \overset{(8)}{\eta_x\frac{\partial Z}{\partial \eta}}\right] + \left[\xi_y\frac{\partial}{\partial \xi}\left(\frac{1}{2R_T}\right) + \eta_y\frac{\partial}{\partial \eta}\left(\frac{1}{2R_T}\right)\right]$$
$$\times\left[\overset{(9)}{\xi_y\frac{\partial Z}{\partial \xi}} + \overset{(10)}{\eta_y\frac{\partial Z}{\partial \eta}}\right] + \text{RST} + \text{AXRST} \tag{3}$$

The last terms on both the left- and right-hand sides are particular to the axisymmetric formulation; all other terms appear in both the two-dimensional and the axisymmetric formulation. The coefficients α, β, ξ_x, ξ_y, etc., are generated

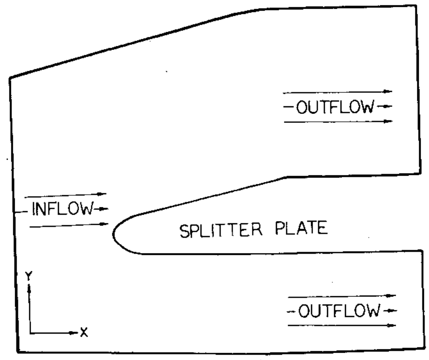

Fig. 1 Geometry of flow region in physical plane.

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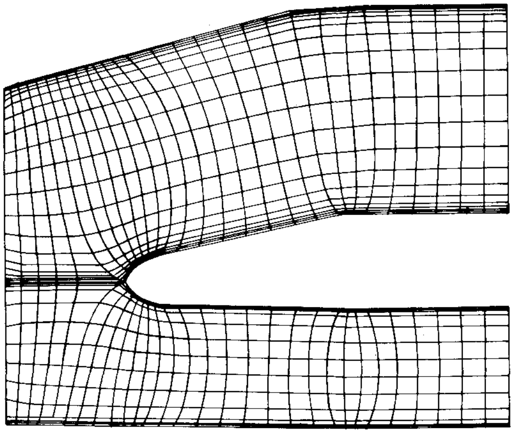

Fig. 2 Body-fitted coordinate system as viewed in physical plane.

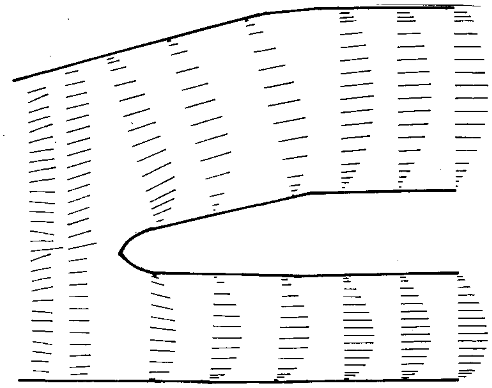

Fig. 3 Velocity plot, axisymmetric, turbulent flow; $R = 750{,}000$.

as part of the solution of Eqs. (2) and, thus, are known quantities. R_{eff} is the Reynolds number based on the sum of the kinematic and eddy viscosity, and R_T is the Reynolds number based on the eddy viscosity. The quantities RST and AXRST represent those terms remaining after substitution of the eddy viscosity model for the Reynolds stresses, but which cannot be combined to form vorticity. Similar transport equations exist for the two parameters in the eddy viscosity model; a complete set of the required equations can be found in Ref. 4.

Separate codes are written for the solution of Eqs. (2) and (3). Thus, any changes in the flow problem involving the geometry or the coordinate system, two-dimensional or axisymmetric, are contained within the transformation program. Therefore, geometric considerations are removed from the solution of the flow equations, and the flow code can be used for a wide range of problems without need of modification.

Results

All equations are solved by finite difference techniques.[4] The 10 indicated terms in Eq. (3) are upwind differenced, with the remaining terms central differenced; the resulting equations are solved using the ADI technique. It was found that all similar first-order terms in the stream function equation and the eddy parameter transport equations also required individual upwind differencing.

Figure 3 shows the velocity field for the axisymmetric, turbulent configuration at the Reynolds number of 750,000, based on inlet channel width. Solutions were found for both the two-dimensional and the axisymmetric flows over a Reynolds number range of 1350 to 750,000. In each case, a solution was found for a low Reynolds number and this was used as a starting solution for a higher Reynolds number. This considerably reduced the time needed to obtain solutions at high Reynolds numbers. Steady state was taken to be when the maximum change in the velocity field from one step to the next was less than 10^{-6}. For the initial solution, this represented 150 time steps, requiring 45 min on an Itel AS/6 computer. Using a converged solution at a low Reynolds number as the starting condition for a higher Reynolds number produced conveyed solutions in 15 min of computer time. The solutions were obtained for a 30×38 grid. Standard procedure would be to check the solutions by doubling the number of grid points and resolving the problem. Unfortunately, this is not easy to do, since the algorithm requires simultaneous use of all metric data. The 30×38 grid required almost the full core of the Itel AS/6 system; thus, it was not possible to increase the grid size without extensive reprogramming for external storage and data transfer.

The study indicates that:

1) First-order terms must be separately upwind differenced to form stable solutions, regardless of whether the term is a true velocity term of a geometrically generated term created by the transformation.

2) Axisymmetric formulation does not increase programming complexity, but does greatly increase the flexibility of the code.

3) The necessity of having all metric data in core means that even problems with a moderate number of grid points will tend to require large core storage.

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